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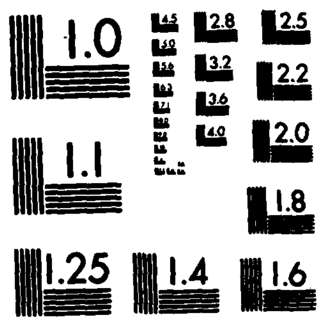
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ABSTRACT

The extended Kalman Filter (EKF), in the continuous time version, has to be interpreted in the Itô sense. Therefore, processing of the (EKF) algorithm is much more involved than has appeared to be the case. An additional $\frac{n^2 (n+1)}{2}$ differential equations have to be processed when the EKF is properly interpreted.

A NOTE ON THE EXTENDED KALMAN FILTER

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ABSTRACT

The extended Kalman Filter (EKF), in the continuous time version, has to be interpreted in the Itô sense. Therefore, processing of the (EKF) algorithm is much more involved than has appeared to be the case. An additional $\frac{n^2 (n+1)}{2}$ differential equations have to be processed when the EKF is properly interpreted.

PRESENTATION

Consider the continuous nonlinear stochastic dynamic system and observations given by

$$d\underset{\sim}{x}(t) = \underset{\sim}{f}(\underset{\sim}{x}(t),t)\ dt + G(t)\ d\underset{\sim}{w}(t),\quad x(t_o) = X_o \tag{1}$$

And

$$d\underset{\sim}{z}(t) = \underset{\sim}{h}(\underset{\sim}{x}(t),\ t)\ dt + d\underset{\sim}{v}(t) \tag{2}$$

Where

$\underset{\sim}{x}(t) \epsilon R^n$ is an 'n' dimensional state vector.

$\underset{\sim}{f}(\underset{\sim}{x}(t),t)$ is an 'n' dimensional vector valued function.

$\underset{\sim}{x}_o \epsilon R^n$ is an 'n' dimensional Gaussian random vector with mean $\underset{\sim}{\bar{x}}_o$, and covariance P_o.

$\underset{\sim}{z}(t) \epsilon R^m$ is an 'm' dimensional observations vector.

$\underset{\sim}{h}(\underset{\sim}{x}(t),t)$ is an 'm' dimensional vector valued function.

$\underset{\sim}{w}(t) \epsilon R^n$ is an 'n' dimensional Wiener process with zero mean and covariance Q(t).

$\underset{\sim}{v}(t) \epsilon R^n$ is an 'n' dimensional Wiener process with zero mean and covariance R(t).

Under the assumption that x_o, $w(t)$, and $v(t)$ are mutually independent of each other, and $R(t)$ considered to be positive definite matrix, the widely used Extended Kalman Filter (EKF) is given by the following set of equations [1]

$$d\hat{\underset{\sim}{x}}(t) = \underset{\sim}{f}(\hat{\underset{\sim}{x}}(t),t)\,dt + P(t)\,H^T(\hat{\underset{\sim}{x}}(t),t)R^{-1}(t)\left[d\underset{\sim}{z}(t) - \underset{\sim}{h}(\hat{\underset{\sim}{x}}(t),t)\,dt\right], \quad \hat{\underset{\sim}{x}}(t_o) = \bar{\underset{\sim}{x}}_o \tag{3}$$

and

$$\frac{d}{dt}P(t) = F(\hat{\underset{\sim}{x}}(t),t)\,P(t) + P(t)F^T(\hat{\underset{\sim}{x}}(t),t) + G(t)Q(t)G^T(t) - P(t)\,H^T(\hat{\underset{\sim}{x}}(t),t)\,R^{-1}(t)\,H(\hat{\underset{\sim}{x}}(t),t)P(t),$$

$$P(t_o) = P_o \tag{4}$$

Where

$$H(\hat{\underset{\sim}{x}}(t),t) \triangleq \left[\frac{\partial h_\ell(\underset{\sim}{x}(t),t)}{\partial\, x_j(t)}\bigg|_{\underset{\sim}{x}(t) = \hat{\underset{\sim}{x}}(t)}\right] \tag{5}$$

and

$$F(\hat{\underset{\sim}{x}}(t),t) \triangleq \left[\frac{\partial f_\ell(\underset{\sim}{x}(t),\ t)}{\partial x_j\ (t)}\bigg|_{\underset{\sim}{x}(t)=\hat{\underset{\sim}{x}}(t)}\right] . \tag{6}$$

It is clear that the matrices $H(\hat{\underset{\sim}{x}}(t),t)$ and $F(\hat{\underset{\sim}{x}}(t),t)$ as given by equations (5) and (6) are state estimate dependent. Hence, also the covariance matrix is state estimate dependent, and a more accurate description of the (EKF) is as follows:

$$d\hat{\underset{\sim}{x}}(t)=f(\hat{\underset{\sim}{x}}(t),t)dt+K(\hat{\underset{\sim}{x}}(t),t)\left[d\underset{\sim}{z}(t)-\underset{\sim}{h}(\hat{\underset{\sim}{x}}(t),t)dt\right],$$
$$\hat{\underset{\sim}{x}}(t_o)=\bar{\underset{\sim}{x}}_o \tag{7}$$

$$\frac{d}{dt}P(\hat{\underset{\sim}{x}}(t),t)=F(\hat{\underset{\sim}{x}}(t),t)P(\hat{\underset{\sim}{x}}(t),t)+P(\hat{\underset{\sim}{x}}(t),t)F^T(\hat{\underset{\sim}{x}}(t),t)+$$

$$G(t)Q(t)G^T(t)-$$

$$P(\hat{\underset{\sim}{x}}(t),t)H^T(\hat{\underset{\sim}{x}}(t),t)R^{-1}(t)H(\hat{\underset{\sim}{x}}(t),t)P(\hat{\underset{\sim}{x}}(t)\ \ t),P(t_o)=P_o \quad (8)$$

Where

$K(\hat{\underset{\sim}{x}}(t),t)=P(\hat{\underset{\sim}{x}}(t),t)H^T(\hat{\underset{\sim}{x}}(t),t)R^{-1}(t)$ is the filter's gain matrix.

Now, it is easy to recognize that equation (7) is an Itô stochastic differential equation. Hence, it should be integrated in the Itô sense. Therefore, whenever numerical simulations of the (EKF) are carried out, equation (7) has to be transformed into an equivalent differential equation in an ordinary sense such as the Stratonovich sense. The Stratonovich equivalent of (7) is obtained by modifying the $i^{\underline{th}}$ component

$f_i(\hat{\underset{\sim}{x}}(t),t)$ by the following quantity

$$-\frac{1}{2} \sum_{\ell=1}^{n} \sum_{j=1}^{m} K_{\ell j}\ (\hat{\underset{\sim}{x}}(t),t)\ \frac{\partial K_{ij}(\hat{\underset{\sim}{x}}(t),t)}{\partial\ \hat{x}_\ell(t)}.$$

Therefore, for the completeness of the EKF, the expression

$$\frac{\partial K_{ij}(\hat{\underset{\sim}{x}}(t),t)}{\partial\ \hat{x}_\ell(t)} \quad \text{for all } i,\ j, \text{ and } \ell$$

has to be included. Denote $\frac{\partial K\ (\hat{\underset{\sim}{x}}(t),t)}{\partial\ \hat{x}_i(t)}$ by

$K_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)$, and $\frac{\partial\ P(\hat{\underset{\sim}{x}}(t),t)}{\partial \hat{x}_i}$ by $P_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)$, then

$$K_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)=\Bigg[P(\hat{\underset{\sim}{x}}(t),t)\frac{\partial H^T(\hat{\underset{\sim}{x}}(t),t)}{\partial \hat{x}_i} + \frac{\partial P(\hat{\underset{\sim}{x}}(t),t)}{\partial \hat{x}_i}\ H^T(\hat{\underset{\sim}{x}}(t),t)\Bigg] R^{-1}(t) \tag{9}$$

And for $P_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)$, assuming that $P(\hat{\underset{\sim}{x}}(t),t)$

is continuous in both arguments, we have

$$\begin{aligned}\frac{d}{dt}\hat{P}_{x_i}\ (\hat{\underset{\sim}{x}}(t),t)=&F(\hat{\underset{\sim}{x}}(t),t)P_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)+F_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)\ P(\hat{\underset{\sim}{x}}(t),t)+\\ &P_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)F^T(\hat{\underset{\sim}{x}}(t),t)+P(\hat{\underset{\sim}{x}}(t),t)F^T_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)\\ &-P_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)\ H^T\ (\hat{\underset{\sim}{x}}(t),t)R^{-1}(t)H(\hat{\underset{\sim}{x}}(t),t)P(\hat{\underset{\sim}{x}}(t),t)\\ &-P(\hat{\underset{\sim}{x}}(t),t)\ H^T_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t)\ R^{-1}(t)H(\hat{\underset{\sim}{x}}(t),t)P(\hat{\underset{\sim}{x}}(t),t)\\ &-P(\hat{\underset{\sim}{x}}(t),t)H^T(\hat{\underset{\sim}{x}}(t),t)\ R^{-1}(t)\ H_{\hat{x}_i}(\hat{\underset{\sim}{x}}(t),t)\ P(\hat{\underset{\sim}{x}}(t),t)\\ &-P(\hat{\underset{\sim}{x}}(t),t)\ H^T(\hat{\underset{\sim}{x}}(t),t)\ R^{-1}(t)\ H(\hat{\underset{\sim}{x}}(t),t)\ P_{\hat{x}_i}\ (\hat{\underset{\sim}{x}}(t),t),\end{aligned}$$

$$P_{\hat{x}_i}\ (t_o) = 0 \tag{10}$$

for every $i = 1, 2, ...,n$

where

$$H_{\hat{x}_i}(\hat{\underset{\sim}{x}}(t),t)=\frac{\partial\ H(\hat{\underset{\sim}{x}}(t),t)}{\partial \hat{x}_i}\ \triangleq\ \frac{\partial}{\partial \hat{x}_i}\left[\frac{\partial h_\ell(\underset{\sim}{x}(t),t)}{\partial x_j}\Bigg|_{\underset{\sim}{x}(t)=\hat{\underset{\sim}{x}}(t)}\right]$$

$$= \left[\frac{\partial^2 h_\ell(\underset{\sim}{x}(t),t)}{\partial x_i \, \partial x_j} \bigg|_{\underset{\sim}{x}(t)=\hat{\underset{\sim}{x}}(t)} \right] \quad (11)$$

and

$$F_{\hat{x}_i}(\hat{\underset{\sim}{x}}(t),t) = \frac{\partial F(\hat{\underset{\sim}{x}}(t),t)}{\partial \hat{x}_i} \triangleq \frac{\partial}{\partial \hat{x}_i} \left[\frac{\partial f_\ell(\underset{\sim}{x}(t),t)}{\partial x_j} \bigg|_{\hat{\underset{\sim}{x}}(t)=\underset{\sim}{x}(t)} \right]$$

$$= \left[\frac{\partial^2 f_\ell(\underset{\sim}{x}(t),t)}{\partial x_i \, \partial x_j} \bigg|_{\underset{\sim}{x}(t)=\hat{\underset{\sim}{x}}(t)} \right] . \quad (12)$$

Conclusion:

The continuous time EKF has to be interpreted in the Itô sense. Therefore, the actual computational requirement for implementing the EKF is increased. An extra n matrix differential equations for $P_{\hat{x}_i}(\hat{\underset{\sim}{x}}(t),t)$, $i=1,2,\ldots n$ are coupled to the differential matrix Ricatti equation for $P(\hat{\underset{\sim}{x}}(t),t)$. Also, the nonlinearities $f(\underset{\sim}{x}(t),t)$ and $h(\underset{\sim}{x}(t),t)$ are assumed to have mixed second order partial derivatives with respect to the components of $\underset{\sim}{x}(t)$. Of course, if an integration routine that conforms with the Itô definition of stochastic integral is used in processing the (EKF) algorithm, then it is sufficient to process equations (7) and (8). Among the widely used integration routines, the rectangular routine is the only one that conforms with the Itô definition of the stochastic integral. The discussion in this note applies in a similar way to any of the many suboptimal filters whose gain and, or covariance is estimate dependent.

In practical implementation, the filter equations (3) and (4) are discretized. Usually the time increments are taken to be small enough to assure acceptable accuracy. In order to conform with Itô's definition of stochastic integral, the system matrices $F(\hat{x}(t),t)$ and $H(\hat{x}(t),t)$ should be held at their values corresponding to the beginning of each particular increment.

References

1 A. H. Jazwinski, *Stochastic Processes And Filtering Theory*, Academic Press, New York, 1970.

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